Calculations about a lever

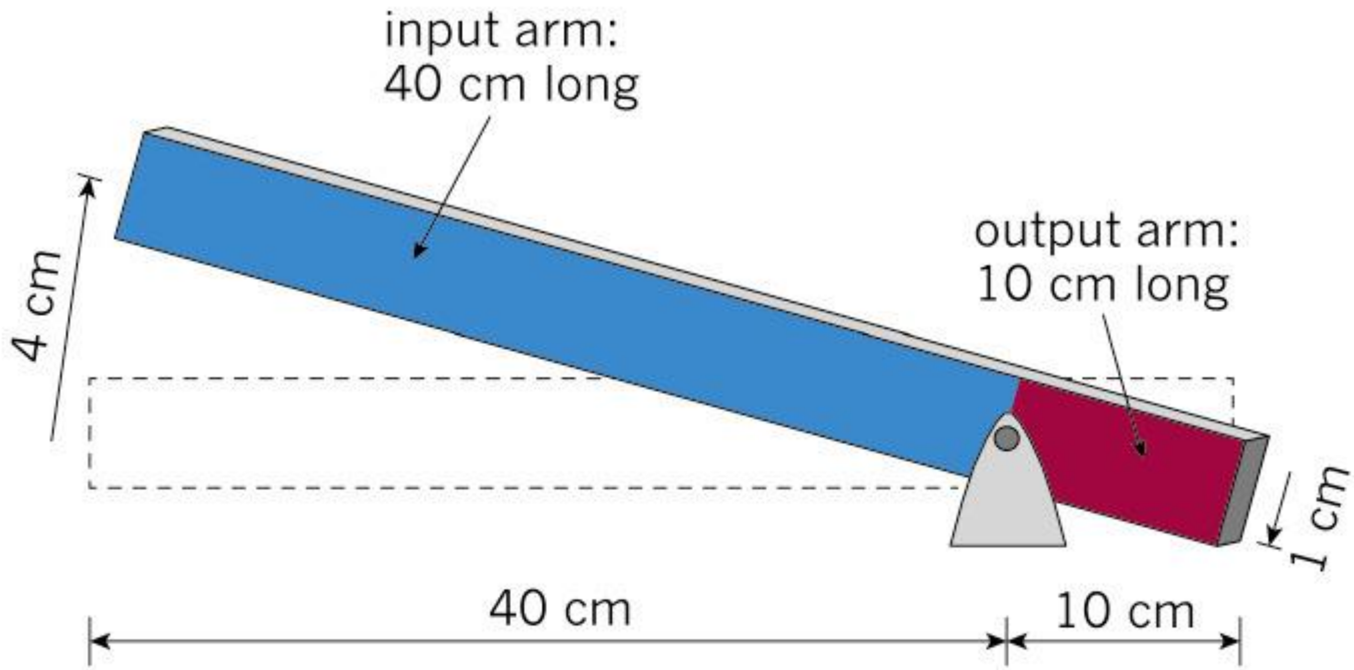

Figure 2: This lever has two arms: an input arm in blue and an output arm in red.

On the lever above, the input arm is 40 cm long and the output arm is 10 cm long. The input arm on this lever has been moved up by 4 cm. Study the picture and then answer the questions.

The ratio of input arm and output arm in levers

If the input arm is 400 cm long and the output arm is 100 cm long, then the output distance will always be:

$$\text{output distance} = \frac{100}{400} \times \text{input distance.}$$

If you lifted this lever by 20 cm, then:

$$\text{Output distance} = \frac{100}{400} \times \text{input distance} = \frac{100}{400} \times 20\text{ cm} = \frac{1}{4} \times 20\text{ cm} = 5\text{ cm.}$$

This lever gives you a mechanical advantage because the input distance is larger than the output distance. We know that a lever with a larger input distance and a smaller output distance will give a mechanical advantage, so we can say that:

Input distance ÷ output distance
= length of input arm ÷ length of output arm
= mechanical advantage (MA)

But we also know that:
Input distance ÷ output distance
= output force ÷ input force
= mechanical advantage (MA).

The ratio input arm distance: output arm distance is the same as the ratio output force: input force, and this is the mechanical advantage.

A lever with an input arm 400 cm long and the output arm 100 cm long will give a mechanical advantage of:

$$MA = \frac{\text{input arm distance}}{\text{output arm distance}} = 400 \div 100 = 4.$$

This means that the output force will always be four times larger than the input force; and the input force will always be four times less than the output force.

Consider the force needed to keep a weight of 20 kg from falling due to gravity. If this weight is on the output side of the lever discussed above, then what weight is needed on the input side of the lever to balance it?

Weight on input side = $\frac{1}{4} \times 20$ kg = 5 kg

Mechanical advantage

- $MA = \frac{\text{output force}}{\text{input force}}$
- $= \frac{\text{input arm distance}}{\text{output arm distance}}$
- $\text{output force} = MA \times \text{input force}$
- $\text{input force} = \frac{\text{output force}}{MA}$

Calculations about a lever

Look at the lever in Figure 3. The lever is pushed down to crush a can.

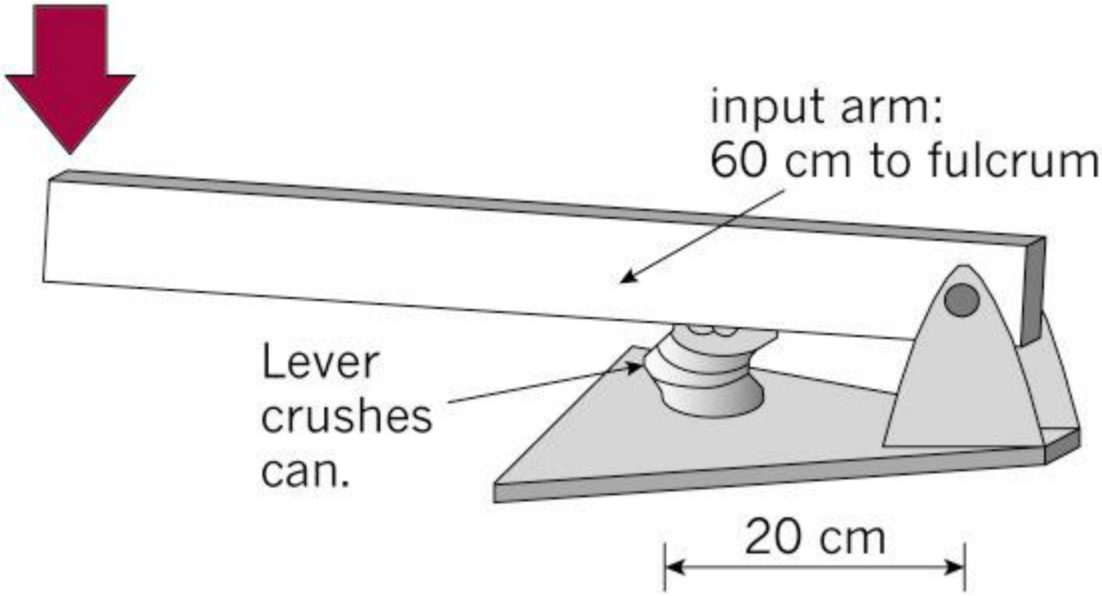

Figure 3: A lever crushing a can

1. How do you know that this lever will crush a can more easily than by hand?

2. With the can in the position shown, calculate the mechanical advantage that this lever will give.

3. If you need an output force of 20 "units of force" to crush the can, how much input force do you need?

4. The designer decides to make it even easier to crush the can. She moves the can closer to the fulcrum. This reduces the output arm to 15 cm. Recalculate the mechanical advantage of the lever.

5. Recalculate the new input force needed to crush the can with an output force of 20 units.

Calculate the distance advantage of a lever

Look at the lifting system in Figure 4. It uses a hydraulic cylinder for the input force. It is a system that could be used for lifting an engine out of a motorcar.

The lifting lever at the top is a third-class lever, because the input is between the fulcrum and the output.

A third-class lever always gives a distance advantage. It never gives a mechanical advantage.

1. How long is the input arm on this lever?

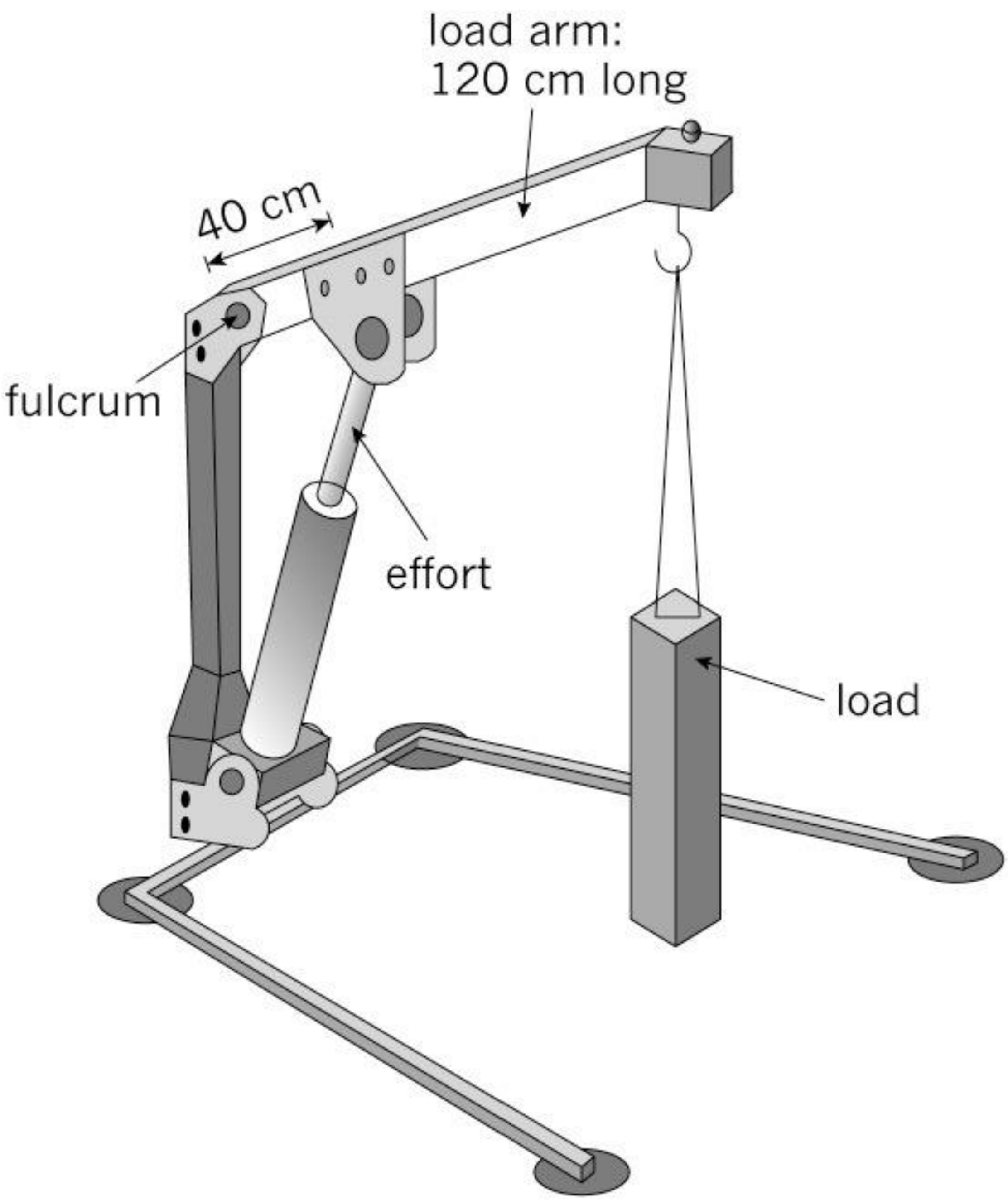

Figure 4: A lifting system

2. How long is the output arm?

3. Calculate the mechanical advantage that this lever gives.

4. Explain what this MA value tells you about the output and input forces.

5. A person wants to use this system to lift an engine out of a car. He needs the engine to be lifted by 90 cm. How far will the hydraulic cylinder at the input need to move for the engine to be lifted 90 cm at the output?

6. If the system is designed to lift objects by 180 cm, how far does the hydraulic cylinder need to move at the input?

Calculate the speed advantage of gears

The gear ratio

The gear ratio, which is also called the speed ratio or sometimes the velocity ratio, tells you how the speed of a driven gear will be changed by a driver gear.

Look at the two meshed gears in Figure 5. The driver or input gear has 5 teeth. The driven or output gear has 10 teeth.

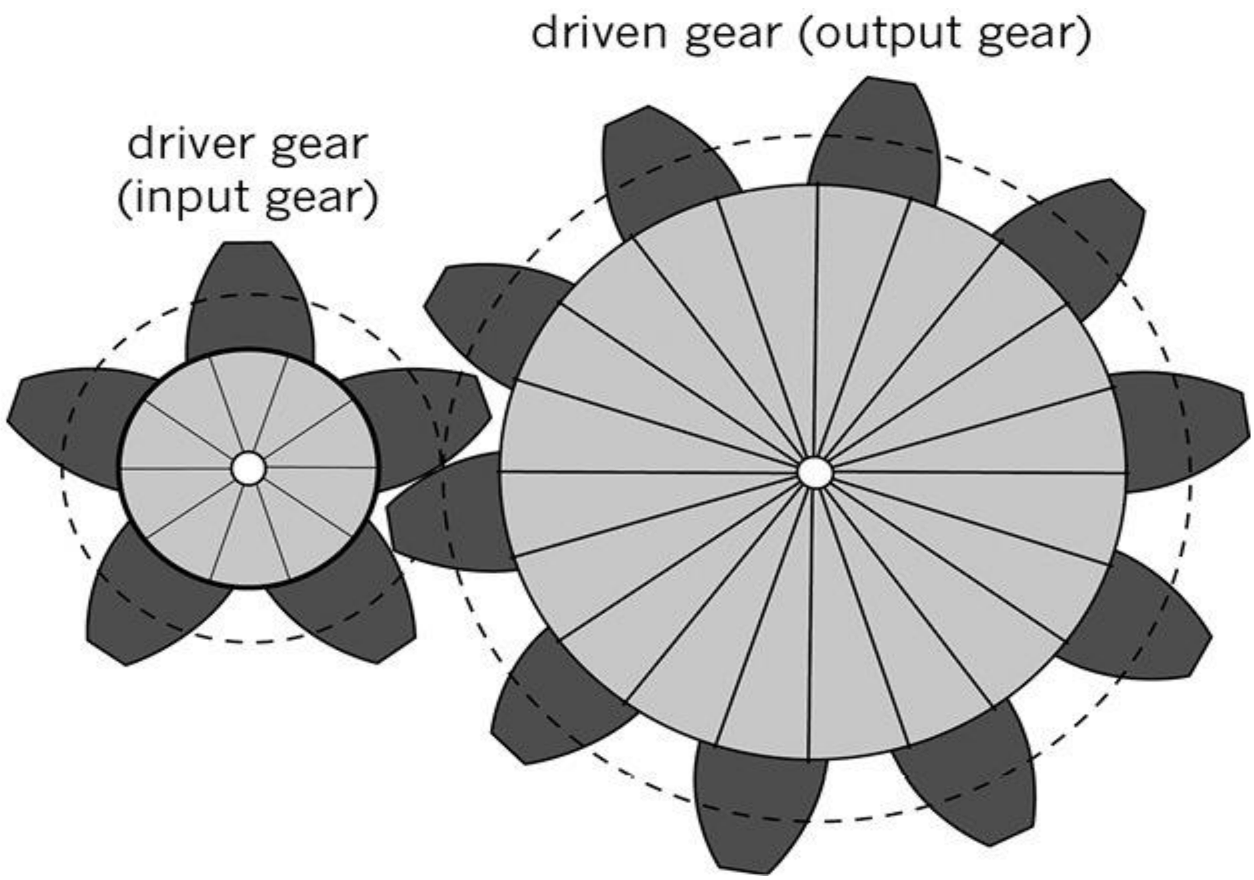

Figure 5

- If the driver moves one revolution, it pushes the 5 teeth on the driven gear.
- If the driver gear moves by 2 revolutions, then 10 of the driven gear's teeth are moved. So the driven gear moves 1 revolution. Two driver revolutions give 1 driven gear revolution. So the turning speed of the driven gear is ½ the turning speed of the driver gear. The speed ratio, which is the same as the gear ratio, is1: 2 or ½.

- If the driver gear revolves 10 times, then the driven gear will only revolve 5 times.

$$\text{gear ratio} = \frac{\text{rotational speed of input axle}}{\text{rotational speed of output axle}}$$

$$= \frac{\text{number of teeth on output gear}}{\text{number of teeth on input gear}} = \frac{10}{5} = 2$$

Calculations using the gear ratio

The speed of a turning wheel is measured in revolutions per minute, or **rpm**. So if the driver gear is turning around twice every second, it has a speed of 2 rpm. A speed of 2 rpm on this system will give a speed of 1 rpm on the driven gear.

rpm stands for "revolutions per minute".

The gear ratio can be used to work this out:

$$\text{Driver gear speed} = (\text{gear ratio}) \times (\text{driven gear speed})$$
$$= \frac{1}{2} \times 2 \text{ rpm}$$
$$= 1 \text{ rpm.}$$

The speed ratio and mechanical advantage

If a gear system gives a **speed advantage** because of its gear ratio, then it will give you a mechanical disadvantage. This means that if a driven gear revolves faster than its driver gear, it gives less turning output force to the machine.

If a driven gear revolves slower than its driver gear, it gives more turning output force to the machine.

Speed advantage

When a driven (input) gear makes the driver (output) gear rotate faster, then the gear system gives a speed advantage.

Calculations

1. The gear system in Figure 5 has 5 teeth on the driver gear and 10 teeth on the driven gear. Calculate the rpm of the driven gear if the driver gear rotates at 1 500 rpm.

2. If a driver gear has 15 teeth and a driven gear has 60 teeth, calculate the gear ratio.

3. Consider a gear system where the driver gear has 25 teeth and the driven gear has 15 teeth.

(a) If the driver gear rotates at 100 rpm, calculate the speed of the driven gear